Integers

Natural Numbers

Rational Numbers

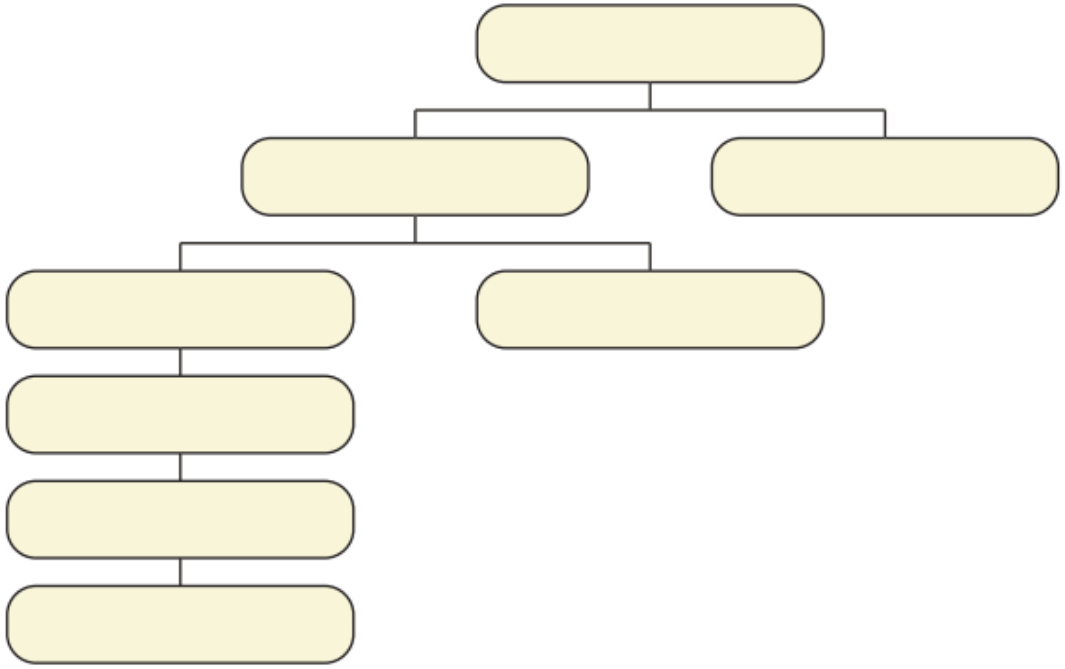
Whole Numbers

Real Numbers

Complex Numbers

Irrational Numbers

Imaginary Numbers



The Imaginary Unit *i*

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Not all quadratic equations have real-number solutions. For example, $x^2 = -3$ has no real-number solutions because the square of any real number is never a negative number.

To overcome this problem, mathematicians created an expanded system of numbers using the **imaginary unit *i***, defined as $i = \sqrt{-1}$. Note that $i^2 = -1$. The imaginary unit i can be used to write the square root of *any* negative number.

KEY IDEA

The Square Root of a Negative Number

Property	Example
1. If r is a positive real number, then $\sqrt{-r} = \sqrt{-1}\sqrt{r} = i\sqrt{r}$.	$\sqrt{-3} = \sqrt{-1}\sqrt{3} = i\sqrt{3}$
2. By the first property, it follows that $(i\sqrt{r})^2 = i^2 \cdot r = -r$.	$(i\sqrt{3})^2 = i^2 \cdot 3 = -1 \cdot 3 = -3$

Finding Square Roots of Negative Numbers

Find the square root of each number.

$$\sqrt{-25}$$

Find the square root of each number.

$$\sqrt{-72}$$

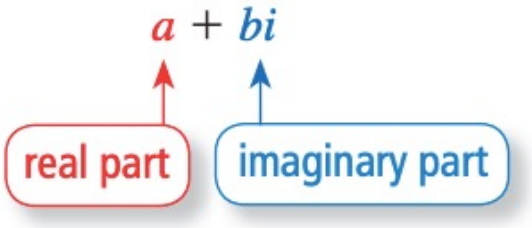
Find the square root of each number.

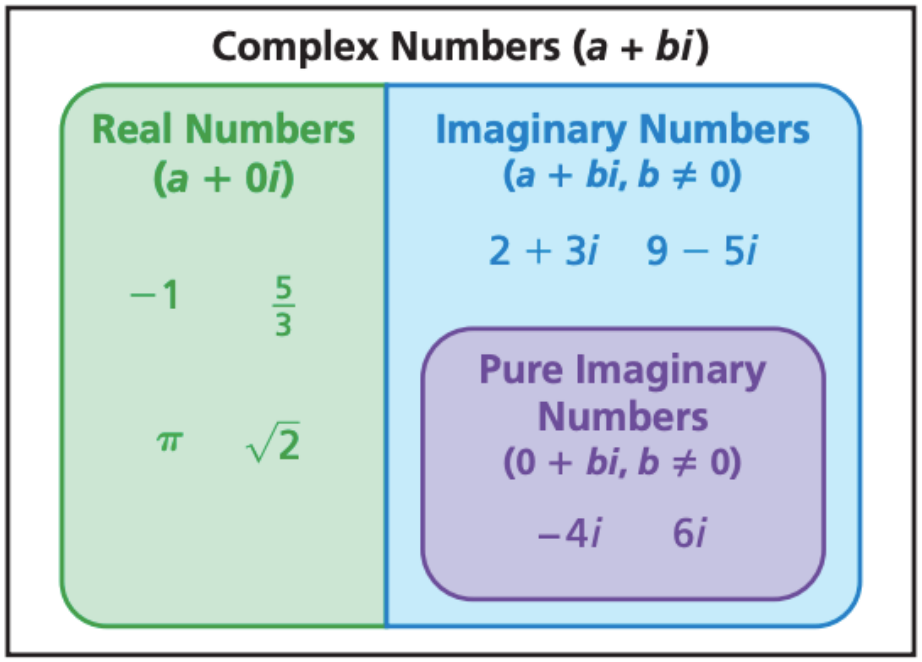
$$\sqrt{-98}$$

Find the square root of each number.

$$-5\sqrt{-9}$$

A **complex number** written in *standard form* is a number $a + bi$, where a and b are real numbers. The number a is the *real part*, and the number bi is the *imaginary part*.





Equality of Two Complex Numbers

Two complex numbers $a + bi$ and $c + di$ are equal if and only if $a = c$ and $b = d$.

Find the values of x and y that satisfy the equation $2x - 7i = 10 + yi$.

Find the values of x and y that satisfy the equation.

$$x + 3i = 9 - yi$$

Find the values of x and y that satisfy the equation.

$$5x + 4i = 20 + 2yi$$

Find the values of x and y that satisfy the equation.

$$9 + 4yi = -2x + 3i$$

Operations with Complex Numbers

KEY IDEA

Sums and Differences of Complex Numbers

To add (or subtract) two complex numbers, add (or subtract) their real parts and their imaginary parts separately.

Sum of complex numbers: $(a + bi) + (c + di) = (a + c) + (b + d)i$

Difference of complex numbers: $(a + bi) - (c + di) = (a - c) + (b - d)i$

Add or subtract. Write the answer in standard form.

$$(8 - i) + (5 + 4i)$$

Add or subtract. Write the answer in standard form.

$$(9 - i) + (-6 + 7i)$$

Add or subtract. Write the answer in standard form.

$$5 + (-9 + 3\,i\,) + 6i$$

Add or subtract. Write the answer in standard form.

$$(7 - 6i) - (3 - 6i)$$

Add or subtract. Write the answer in standard form.

$$(3 + 7i) - (8 - 2i)$$

Add or subtract. Write the answer in standard form.

$$-4 - (1 + i) - (5 + 9i)$$