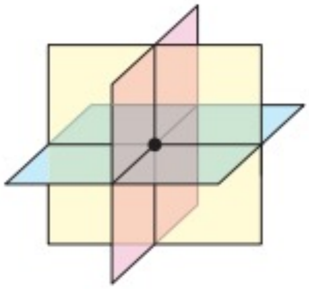
Exactly One Solution

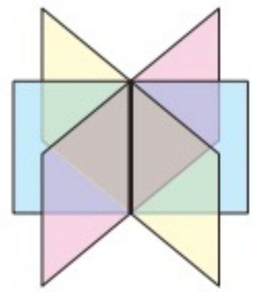
The planes intersect in a single point, which is the solution of the system.



Infinitely Many Solutions

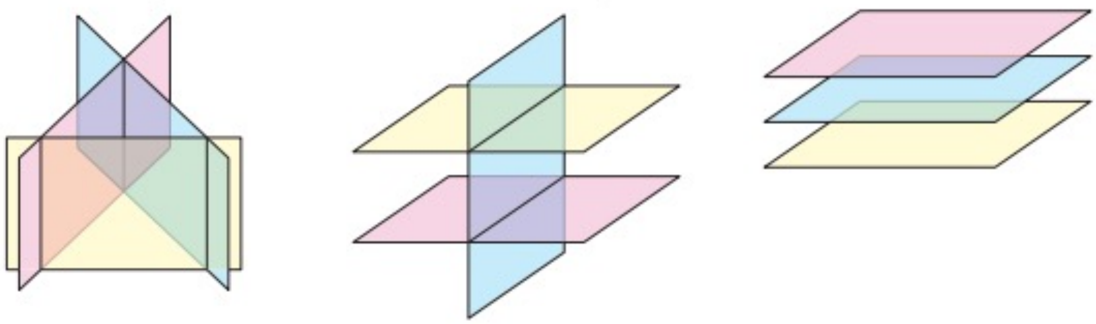
The planes intersect in a line. Every point on the line is a solution of the system.

The planes can also be the same plane. Every point in the plane is a solution of the system.



No Solution

There are no points in common with all three planes.



Solving Systems of Equations Algebraically

The algebraic methods you used to solve systems of linear equations in two variables can be extended to solve a system of linear equations in three variables.

KEY IDEA

Solving a Three-Variable System

Step 1 Rewrite the linear system in three variables as a linear system in two variables by using the substitution or elimination method.

Step 2 Solve the new linear system for both of its variables.

Step 3 Substitute the values found in Step 2 into one of the original equations and solve for the remaining variable.

When you obtain a false equation, such as $0 = 1$, in any of the steps, the system has *no solution*. When you do not obtain a false equation, but obtain an identity such as $0 = 0$, the system has *infinitely many solutions*.

$$4x + 2y + 3z = 12$$
$$2x - 3y + 5z = -7$$
$$6x - y + 4z = -3$$

$$x - 2y + z = -11$$
$$3x + 2y - z = 7$$
$$-x + 2y + 4z = -9$$

$$
\begin{aligned}
5x - 3y + 2z &= 18 \\
-2x + 4y - z &= -11 \\
-3x - 2y + 3z &= 14
\end{aligned}
$$

$$5x + 2y - 4z = -6$$
$$4x - 3y + 2z = 20$$
$$-x + 4y + 6z = 8$$